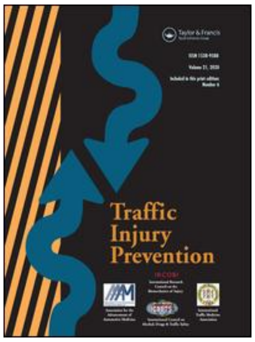

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Traffic Injury Prevention

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/gcpi20

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TRAFFIC INJURY PREVENTION
https://doi.org/10.1080/15389588.2020.1842379

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A penalized-likelihood approach to characterizing bridge-related crashes in New Jersey

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ABSTRACT

Objective: A roadway departure crash is one in which a vehicle crosses an edge line, a centerline, or otherwise leaves the traveled way. These crashes that involve run-off-road and cross-median/centerline head-on collisions tend to be more severe than other crash types. According to the NHTSA Fatality Analysis Reporting System database, a total of 7,833 people perished in crashes involving fixed roadside objects in 2017, accounting for 21 percent of the total number of fatalities in the United States. Several previous studies have reported that rural bridge-related crashes result in more fatalities due to their being mostly the fixed-object crash type. As such, further in-depth investigation of this type of crash is necessary. Due to the lack of a comprehensive database that includes bridge-related crashes and bridge characteristics, identifying the key factors contributing to this type of crash is a challenging task that is addressed in this paper.

Method: Study team gathered and compiled five years (2011–2015) of crash data from the New Jersey crash database and the characteristics of bridges from the Long-Term Bridge Performance portal. A Firth's penalized-likelihood logistic regression model was developed to examine the impact of explanatory variables on crash severity.

Results: Based on the five years (2011–2015) of crash data, significant factors (i.e., driver age, weather conditions, surface conditions, lighting conditions, speed limit, roadway characteristics, and direction of traffic) were identified that affect the severity of bridge-related crashes in Middlesex County, New Jersey.

Conclusion: This model is an appropriate tool for predicting the impact of all the confounding variables on the probability of bridge-related crashes while also considering the rareness of the event. Based on the obtained odds ratio, the various effects of the identified variables are discussed, and recommendations made regarding countermeasures policymakers can establish to reduce the number of these crashes in New Jersey.

ARTICLE HISTORY
Received 27 December 2019
Accepted 22 October 2020

KEYWORDS
Bridge-related crash; fixed-object; odds ratio; Firth's logistic model

Introduction

A roadway departure crash is defined by the Federal Highway Administration (FHWA) as "a crash in which a vehicle crosses an edge line, a centerline, or otherwise leaves the traveled way" (FHWA 2019a). These crashes, which involve run-off-road and cross-median/centerline head-on collisions, tend to be more severe than other crash types (Jalayer and Zhou 2016a, 2016b). According to the FHWA, between year 2015 and 2017, an average of 52 percent of motor vehicle traffic fatalities occurred each year due to roadway departure crashes (FHWA 2019a). Moreover, a total of 7,833 people died in fixed-object crashes (e.g., trees, bridge pier, traffic barriers, and guardrail ends) in 2017, accounting for 21 percent of the total fatalities in the United States (U.S., IIHS 2017). Several previous studies (Jalayer and Zhou 2016a) have reported that bridge-related crashes result in more fatalities due to their being mostly the fixed-object crash type. As such, further in-depth investigation of this type of crash is necessary.

Bridges are an essential component of the U.S. highway system (Mehta et al. 2015). According to the FHWA's Long-Term Bridge Performance (LTBP) program, there are more than 611,000 bridges across the U.S. It should be noted that the physical and operational characteristics of bridges differ significantly from those of roads and highways (Retting et al. 2000; Mehta et al. 2015). Specifically, the traffic lanes on bridges are usually narrower than the approach roads and usually lack shoulders that can safely accommodate vehicles and provide drivers with more space to maneuver and avoid crashes. Notably, bridges have different components such as railings, piers, surface wear, and guardrails that can cause safety hazards

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Associate Editor Clay Gabler oversaw the review of this article.
Supplemental data for this article is available online at https://doi.org/10.1080/15389588.2020.1842379

to vehicles traveling on or under them. The bridge railing provides protection at the edges of structures for traffic and pedestrians. The bridge railing should be strong enough to safely redirect the vehicle without serious snagging or overturning (NYSDOT 2006). According to the Federal Highway Administration (FHWA), the starting point of the guardrail is referred to as the end treatment. To absorb the energy of an impact, one common treatment is to have the head slide down the length of the guardrail (FHWA 2020). Therefore, bridge-related characteristics significantly affect the safety of traffic traveling along and near bridges (Mehta et al. 2015).

Over the past years, several studies have focused on identifying the major contributing factors to bridge-related crashes. Turner (1984) analyzed four years of bridge crash data in Texas and found that narrow bridges (less than 28 ft.) were associated with higher crash rates. In a recent study, Elvik et al. (2019) developed a negative binomial regression model to analyze seven years of data (2010–2016) on crashes that had occurred in or around 6,824 road bridges in Norway. The authors of the study found that traffic volume significantly contributed to the likelihood of bridge crashes. The results of a study conducted by Khan et al. (2009) revealed that surface conditions (ice with a lower coefficient of friction) greatly contributed to the frequency of bridge crashes. Mehta et al. (2015) developed safety performance functions to estimate the expected number of crashes on bridges in Alabama. The results of that study demonstrated that bridge length, percentage of trucks, and Annual Average Daily Traffic (AADT) contributed significantly to bridge-related crashes. Buth et al. (2010) investigated the Minnesota (2002–2006) and Texas (1998–2001) data for 32,934 large-vehicle/fixed-object crashes in which the vehicle had hit a bridge structure. The results showed that lane width, bridge density (number of bridges per mile), and bridge width were all significantly associated with the bridge crashes. Schrock et al. (2012) explored the association between pavement conditions, roadway geometry, and large trucks in fixed-object crashes over bridges. The authors concluded that the injury severities associated with fixed-object crashes that involved bridges (e.g., bridge rails, headwalls, piers, abutments, or guardrails) were 4.93 times higher on a limited access facility such as interstate or freeway. In another study, Sagberg et al. (2020) analyzed seven years of single-vehicle crashes on or close to road bridges in Norway. The results indicated that wider bridges had relatively lower crash risk at entrance compared to exit zones.

Despite the several bridge-related crash studies reported in the literature (Turner 1984; Buth et al. 2010; Schrock et al. 2012; Mehta et al. 2015; Elvik et al. 2019), very few (e.g., Schrock et al. 2012; Sagberg et al. 2020) have only identified or compared the factors associated with the injury severity of bridge-related cashes. Due to the minimal relative number of bridge-related crashes and the lack of a comprehensive database that includes bridge-related crashes and bridge characteristics, identifying the key factors contributing to this type of crash is a challenging task, which we address in this paper. A Firth's penalized-likelihood logistic regression was developed to characterize bridge-related crashes. This model is an appropriate tool for predicting the impact of all the confounding variables on the severity of bridge-related crashes while also considering the rareness of the crash event.

Method

Firth's penalized-likelihood logistic regression

Based on the compiled crash and bridge-related data from the New Jersey Crash Database and Long-Term Bridge Performance (LTBP) portal, three conditions were identified concerning bridge-related crashes, including the rarity phenomenon, unbalanced data, and disproportionality of explanatory variables. The observed crash frequency results indicate that vehicular crashes involving bridges are sporadic events; consequently the sample size is very small. Second, some categories for bridge-related crashes have very low frequency, which can cause computational problems (Hosmer et al. 2013). For example, categories such as speed limit between 36 and 45 mph, vehicle age (unknown group), and median type (other group) have less than 2 crash records. Third, statistical guidelines recommend having at least ten observations per each explanatory variable to construct an effective model and calculate the parameter estimates (Gao and Shen 2007). These issues can limit the applicability of standard logistic regression since it uses maximum likelihood estimation (MLE), which suffers from small-sample bias. In this situation, a penalized-likelihood approach (i.e., Firth's logistic regression) is suggested, which can avoid the shortcomings of MLE (Firth 1993).

Firth's penalized-likelihood logistic regression model is a generalization of MLE models that can effectively avoid the bias that otherwise arises in calculations due to the small sample size and rareness of the event (which is the case in this study). We note that Firth's penalized-likelihood logistic regression model has specifically been employed in the medical sciences. For instance, Mulla et al. (2012) investigated a sample of 138 patients, only 16 of which were diagnosed with preeclampsia, for which computational problems would have arisen in an analysis using MLE methods. Xu et al. (2012) examined a sample size of 67 patients, only 28 of whom had developed hypertension, which is a minimal sample size if using the MLE method. Considering the type of dependent variable in this study (injury crashes vs. property-damage-only crashes), a binary logistic regression is recommended.

Firth introduced a penalized MLE into the binary model that can counteract the bias associated with MLE (Firth 1993). The log-likelihood can be expressed as an exponential family model as follows:

$$l(\beta) = t\beta_n - K(\beta_n),\ n = 1, \ldots, k \tag{1}$$

where β_n is the regression parameter to be estimated, t is the vector of the observed sufficient statistics, and K is the number of parameters estimated. The derivative of the log-

Table 1. Description of explanatory variables (continues variables).

Variable	Min	Mean	Max
Continues variables			
*Ln ADT	5.6	10.5	12.3
Bridge roadway width (m)	6.6	22.1	66.4
Length (m)	6.1	62.8	479.1
Left curb/Sidewalk width (m)	0.0	0.7	3.7
Right curb/Sidewalk width (m)	0.0	0.7	4.3

*Ln: National Logarithm.

likelihood, the score function, is employed to calculate the MLE of parameter β_n as follows:

$$(\beta_n) = l'(\beta_n) = t - K(\beta_n),\ n = 1, \dots, k. \tag{2}$$

The score function is the partial derivative of the log-likelihood function. To penalize the MLE, Firth replaced the score function of the binary model with a modified score function as follows:

$$U(\beta_n)* = U(\beta_n) + \alpha n,\ n = 1, \dots, k \tag{3}$$

where α_n has the n^{th} entry, which is represented as:

$$\alpha n = 1/2\, tr\, [I(\beta)^{-1} \partial I(\beta)/\beta n],\ n = 1.., k \tag{4}$$

where tr is the trace function, and (β) is the Fisher's information matrix, the negative expected value of the second derivative of the log-likelihood. Using this method, the MLE drops to zero. For small samples, Firth's method is more accurate in estimating coefficients and evaluating confidence intervals with respect to coverage probabilities, as compared to the performances of the broad class of generalized linear models (Van der Paal 2014). After calculating the parameter estimates for statistically significant variables, the odds ratio (OR) was calculated as a relative measure of the effect. The OR can be used to better understand the direction and magnitude of the change in the probability of the dependent variable with a one-unit change in the specific variable. That is, when the OR is higher than one, the study group (here, injury crashes) is more likely to have the specific characteristic, as defined in the category, than is the reference category. A similar explanation applies to an OR less than one.

Data

In this study, the required data for analysis were obtained from two primary sources: the New Jersey Crash Database and Long-Term Bridge Performance (LTBP) portal. Unlike other crash types that require only the development of specific filters to prepare the final dataset for analysis, bridge-related crashes require some extra steps prior to analysis to ensure that the dataset has the highest possible accuracy. Specifically, possible bridge-related crashes were identified by defining several filters and then additional steps were taken to verify the actual occurrence of these crashes. We first employed a filter to determine the ROR crashed from the crash database, and then we filtered out single-vehicle crashes. The first filter phase produced several crashes that had not occurred on bridges. For example, by considering the vehicle direction, travel crashes that occurred along a bridge underpass were excluded from further analysis. Visual inspection using Google Street View and Microsoft's Bird's Eye view were used to verify the locations of crashes. After filtering out non-bridge-related crashes via steps mentioned above, our final dataset included 284 single-vehicle, run-off-road crashes involving vehicles striking a bridge-related component that had occurred on 116 public bridges in Middlesex County, New Jersey from 2011 to 2015.

Information regarding the bridge characteristics (i.e., surface wear, bridge railing, guardrail end, bridge roadway width, bridge length, left and right curb/sidewalk widths, median type, and average daily traffic) was obtained from the LTBP portal. This portal includes a variety of information related to the historical conditions and traffic data of bridges from 1992–2016, environmental data such as the number of snowfalls and freeze-thaw cycles, a field-testing database of 80 bridges, and legacy information (FHWA 2019b). The LTBP portal is a web-based platform developed by an FHWA LTBP program that collected all data related to bridge performance. Tables A1 (see online supplement) and Table 1 list the explanatory variables along with their percentages used in this study. When looking at these tables, a few points are worthy of mention. For example, almost 40% of bridge-related crashes during this period occurred during the evening and night hours. Moreover, approximately 60% of bridge-related crashes occurred during autumn and winter and 94% of the drivers had not been impaired.

Results

As previously discussed, for the purpose of this study, Firth's model was found to be more suitable than an MLE model for the analysis of the study variables. To estimate the effect of various confounding factors contributing to the probability of bridge-related crashes, the R software package "logistf" was employed (Heinze et al. 2013). First, a model was fit with all the possible contributing factors and then a backward elimination procedure was adopted, based on the penalized-likelihood ratio test, to identify a model that best agreed with the crash data. A forward selection process produced a similar result. Table 2 shows a summary of the results of the Firth's model, including parameter estimates and their corresponding standard errors and ORs. The ORs are used to better understand the direction and magnitude of the change in the probability of the dependent variable with a one-unit change in the specific variable. That is, when the OR is higher than one, the study group (here, injury crashes) is more likely to have the specific characteristic, as defined in the category, than is the reference category. A similar explanation applies to an OR less than one. To be specific, according to Table 2, the OR higher than one indicated that particular category increases the likelihood of injury-severity crashes compared to the reference group. All of the estimated parameters included in the model are statistically significant at the 95 percent confidence interval. The results of the analysis demonstrated that a variety of factors significantly contributed to injury severity. The results from the best-fitted model indicate that bridge-related injury crashes were more likely to involve a

Table 2. Firth's model results.

Explanatory variable	Firth's model β	S.E.	OR (odds ratio)
Time of day			
Afternoon (12–18) vs Morning (6–12) (ref)	−0.792	0.187	0.453
Driver dender			
Male vs Female (ref)	0.516	0.548	1.675
Weather conditions			
Clear vs Adverse (ref)	0.780	1.07	2.181
Surface conditions			
Dry vs Wet (ref)	−0.942	0.196	0.39
Speed limit			
26 mph to 35 mph vs Less than 25 mph (ref)	−0.837	0.251	0.433
Direction of traffic			
One-way vs Two-way (ref)	0.947	1.588	2.577
Lighting condition			
Dark-not light vs Daylight (ref)	−1.309	0.229	0.27
Roadway characteristic			
Straight vs Curve (ref)	−1.036	0.454	0.355

ref: reference variable.

male driver (OR = 1.67), to occur in clear weather conditions (OR = 2.18), and on one-way roads (OR = 2.57). Similarly, bridge-related injury crashes were less likely to occur in dry surface conditions (OR = 0.39), on straight roads (OR = 0.35), in dark lighting conditions (OR = 0.27), in the afternoon (OR = 0.45), and on facilities with speed limits between 26 mph and 35 mph (OR = 0.43).

Discussion

In this study, considering the lighting conditions, the time of day was categorized into four categories, including morning (6:00–12:00), afternoon (12:00–18:00), evening (18:00–24:00), and night (0:00–6:00). In Table 2, morning and afternoon periods account for about 60% of the total number of bridge-related crashes. The results of the analysis indicate that bridge-related crashes during the afternoon were 0.45 times less likely or probable (less severe) to involve injuries compared with those in the morning. This result is in good agreement with several other studies (PennDOT 2017).

With respect to gender, male drivers (65.8 percent) experienced more bridge-related crashes than female drivers (34.2 percent), which means that male drivers were 1.67 times more likely to be involved in injury-related crashes than females. Several reasons might explain this finding. For example, men drive more miles than women and tend to use more risky behaviors such as driving under the influence of alcohol or drugs and speeding, which results in more severe crashes (Levi et al. 2015; Lawrence and Pace 2017).

According to the estimation results presented in Table 2, weather conditions significantly contributed to the injury severity associated with bridge-related crashes. Specifically, bridge-related crashes in clear weather conditions were 2.18 times more likely to be injury-related crashes. This finding is also in a good agreement with those reported by Christoforu et al. (2012) and Jalayer et al. (2018), which indicates that drivers are more likely to be vigilant regarding their surroundings during adverse weather conditions, which mitigates the severity of injuries.

Similar to weather conditions, the surface condition was also found to significantly contribute to the injury severity of bridge-related crashes. Compared to dry surface conditions, wet surface conditions contributed significantly to the severity of injuries. Specifically, bridge-related crashes in dry surface conditions were 39 percent less likely to result in severe injury. Again, this finding is consistent with other research results (Retting et al. 2000).

In this study, the speed limit was divided into five categories, including less than 25 mph, 26–35 mph, 36–45 mph, 46–55 mph, and faster than 56 mph. The speed-related results show that bridge-related crashes that occurred at speed limits between 26 mph and 35 mph were 0.43 times less likely to result in injuries compared to those that happened at speed limits below 25 mph. No specific reasons were found to explain this phenomenon. Further studies and more data are needed to determine whether this trend persists over a more extended time period.

When comparing injury and property-damage-only bridge-related crashes, the direction of traffic was found to be a statistically significant factor. As shown in Table 2, bridge-related crashes that occurred in one-way traffic were 2.57 times more likely to have injuries than those with two-way traffic.

Lighting conditions also played a significant role in the injury severity of bridge-related crashes. In this study, lighting conditions were classified into four categories: daylight, dawn/dusk, dark-light, and dark-not lit. Based on the compiled data, 56.7 percent of the bridge-related crashes occurred during daylight. It should be noted that darkness, when there is no lighting provided, decreases the severity of crashes (27 percent decrease in the injury crashes) compared to those in daylight conditions. Several factors might explain this finding. For instance, in dark conditions, drivers tend to engage in more risk-compensating behaviors (e.g., driving at lower speeds, paying more attention to the surroundings), which results in crashes with less severe outcomes (Anarkooli and Hosseinlou 2016).

With regard to roadway characteristics, bridge-related crashes that happen on straight road segments are 0.35 times less likely to produce injuries than those that happen along curves. Roadway curvatures are known to reduce available sight distance and decrease vehicle-control capabilities, which increase the probabilities of crashes and injuries (Jalayer and Zhou 2016a).

Based on the findings obtained in this study, several countermeasures and recommendations can be suggested to reduce the occurrence of this type of crash. Behavior-based countermeasures may consider targeting male drivers in particular, given that this driver group was significantly overrepresented by higher OR values. Engineering countermeasures can focus on increasing bridge widths or approach widths. Guiderail upgrades and signage and lighting enhancements are also recommended. Similar to other studies, this study also has some limitations. For example, the data used in this study were from just one county in New Jersey. Incorporating more data from other counties and states would not only bolster the sample size but might also lead to a more comprehensive result that could facilitate the development of better countermeasures and strategies. Another limitation of this study is the inevitable role of human error in the data collection process by police officers, which affects the level of detail and accuracy of the obtained significant variables.

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